The state of the art: Inversions for flows in the solar interior

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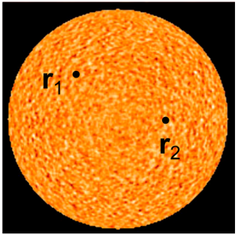
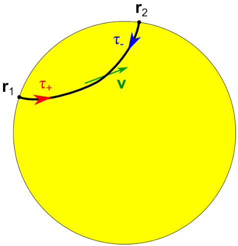
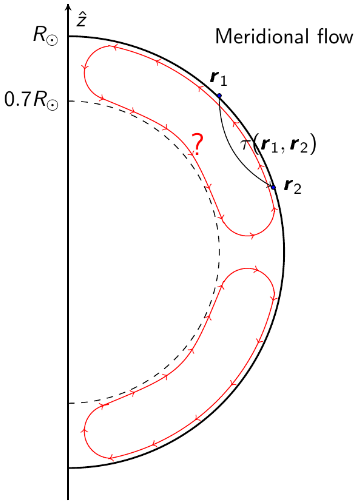
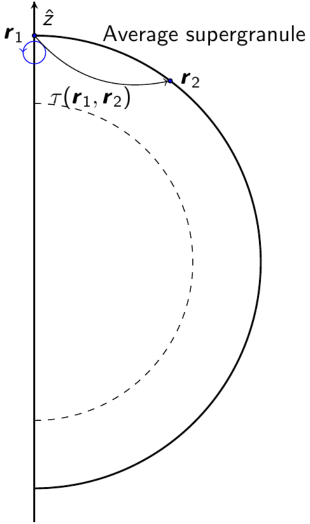
Our mysterious Sun
Tbilisi, Georgia

- Local Helioseismology seeks to probe the internal structure of sun
- Fundamental Quantity: Cross-Covariance
- Travel times tell us about subsurface flows

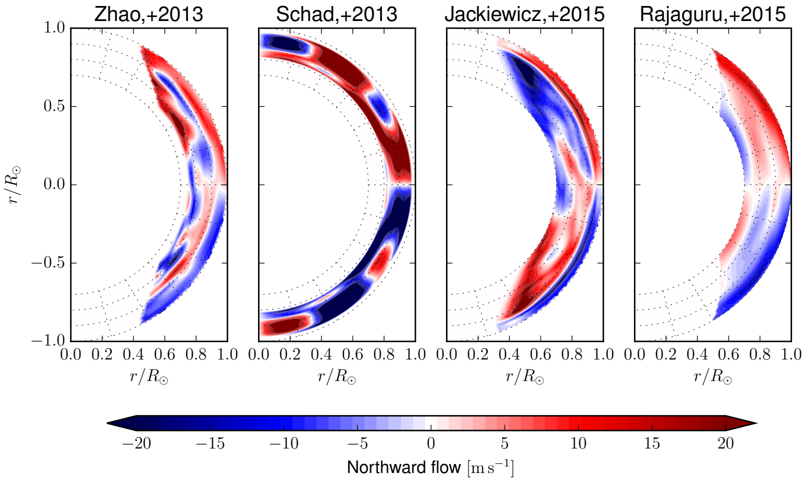





$\hat{z}$
$R_\odot$
Meridional flow
$\boldsymbol{r}_1$
$0.7R_\odot$
?
$\tau(\boldsymbol{r}_1, \boldsymbol{r}_2)$
$\boldsymbol{r}_2$


$\hat{z}$
$\boldsymbol{r}_1$
Average supergranule
$\boldsymbol{r}_2$
$\tau(\boldsymbol{r}_1, \boldsymbol{r}_2)$

- Forward Problem: Develop model and compare results to Obs.
- Inverse Problem: Compute model using Obs. that minimizes difference in results
- Over 20 years of near continuous observations: enough for inversion?



Courtesy of Vincent Böning

- Solve scalar wave equation

$$\mathcal{L}[\psi(\boldsymbol{r},\omega)] = \left(\omega^2 + 2\mathrm{i}\omega\gamma + 2i\omega\boldsymbol{u}\cdot\nabla\right)\psi + c\nabla\cdot\left(\frac{1}{\rho}\nabla(\rho c\psi)\right) = s(\boldsymbol{r}_s)$$

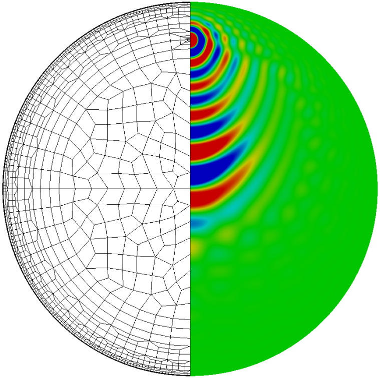
- using Green's functions

$$\mathcal{L}[G(\boldsymbol{r},\boldsymbol{r}_s,\omega)] = \delta(\boldsymbol{r}-\boldsymbol{r}_s)$$

- to obtain the Cross-Covariance

$$C(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega) = \frac{\Pi(\omega)}{4i\omega}\left[G(\boldsymbol{r}_2,\boldsymbol{r}_1,\omega;u) - G^*(\boldsymbol{r}_2,\boldsymbol{r}_1,\omega;-u)\right]$$

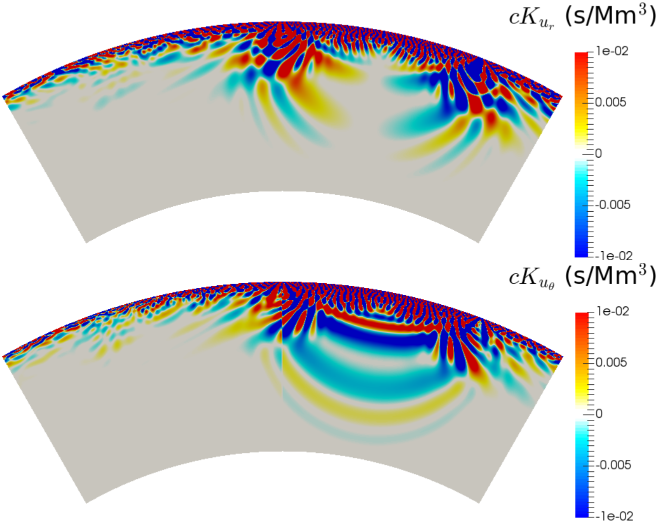
- Solve equation using Finite Elements Method
- Use *Montjoie* Solver
- Assume 2.5D geometry
- Refinement near surface and source



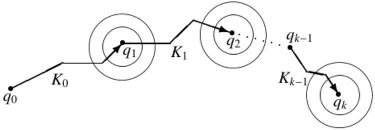
- How are travel times sensitive to perturbations to the model?
- Take First Born Approximation (better then ray theory)
- Develop sensitivity maps for perturbations $\rightarrow$ Kernels

$$\delta q = \int \boldsymbol{K} \cdot \boldsymbol{u} \, \mathrm{d}\boldsymbol{r}$$

$$K_k = -2i\rho \int W_q \omega \left[G(\boldsymbol{r}_2)\partial_k C(\boldsymbol{r}_1) - G^*(\boldsymbol{r}_1)\partial_k C^*(\boldsymbol{r}_2) \right] \mathrm{d}\omega$$


cK_{u_r} (s/Mm^3)
1e-02
0.005
0
-0.005
-1e-02
cK_{u_θ} (s/Mm^3)
1e-02
0.005
0
-0.005
-1e-02

One advantage of our technique: Iterative Inversions Possible



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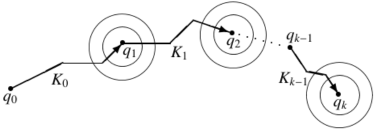

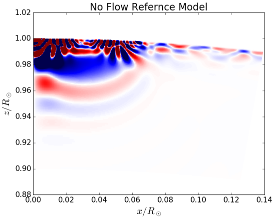

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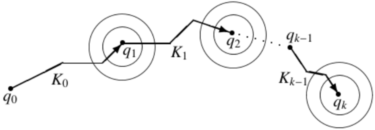

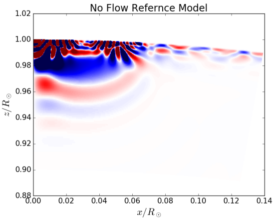

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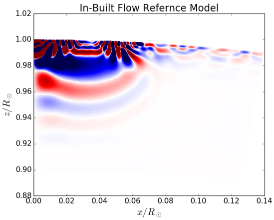

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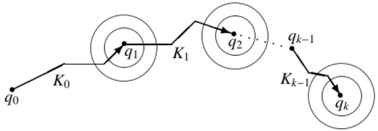
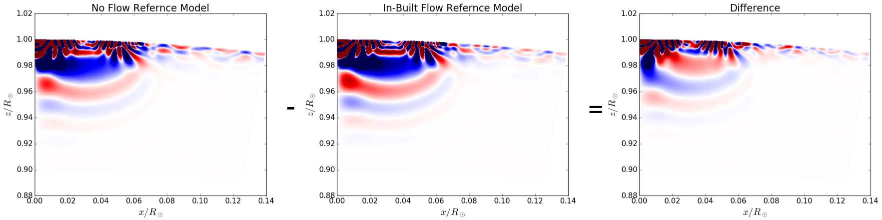
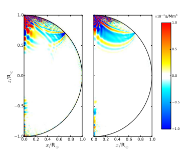
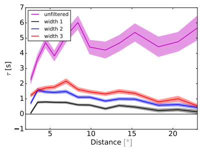


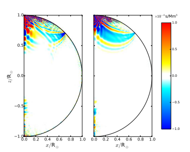


$$\int K(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2)$$



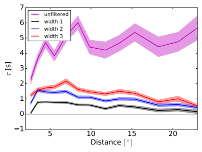
$$\delta q(\boldsymbol{r}_1;\boldsymbol{r}_2) = \int K(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2)$$





Inversions

$\delta q(\boldsymbol{r}_1;\boldsymbol{r}_2) \quad =$



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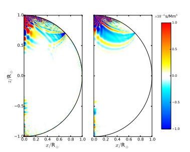
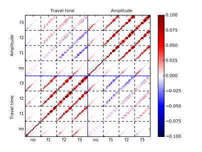
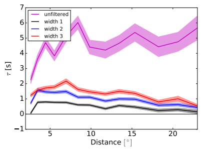
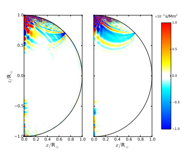


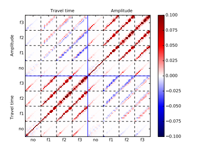
$n(\boldsymbol{r}_1,\boldsymbol{r}_2)$



$$\delta q(\boldsymbol{r}_1;\boldsymbol{r}_2) = \int K(\boldsymbol{x};\boldsymbol{r}_1,\boldsymbol{r}_2) \cdot \boldsymbol{u}(\boldsymbol{x}) + n(\boldsymbol{r}_1,\boldsymbol{r}_2)$$







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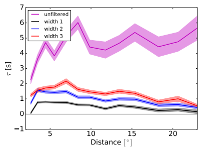

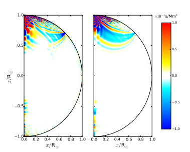

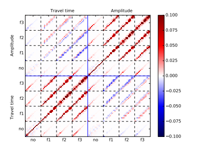

In matrix form

$$\delta q = K\boldsymbol{u} + n,$$

which can be solved for example using the Regularized Least Square (RLS) method which minimizes

$$\|\Lambda^{-1/2}\left(K\boldsymbol{u} - \delta q\right)\|^2 + \alpha\|L\boldsymbol{u}\|^2$$

where Λ is the noise covariance matrix and L is a smoothing matrix. It implies

$$\boldsymbol{u} = (K^*\Lambda^{-1}K + \alpha L^*L)^{-1}K^*\Lambda^{-1/2}\delta q.$$

With all the ingredients, let's try invert for Supergranule!

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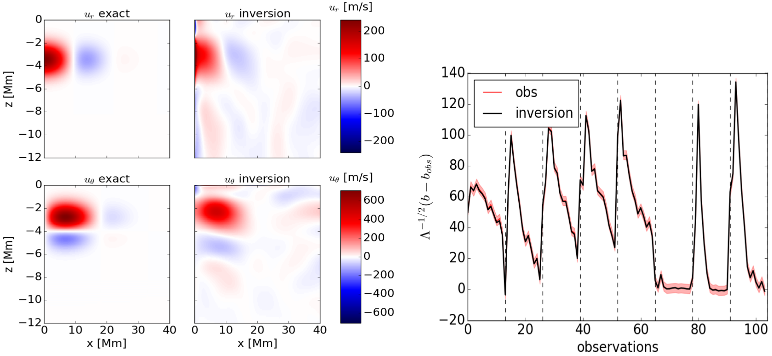

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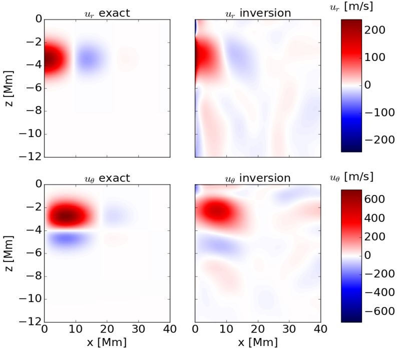

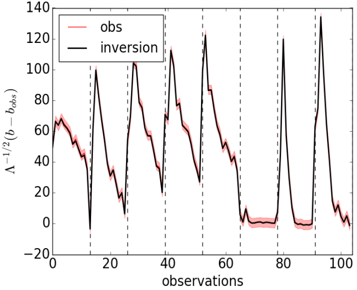

However...

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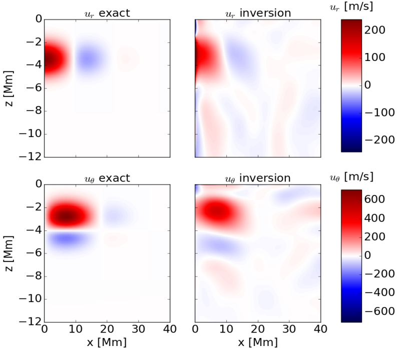

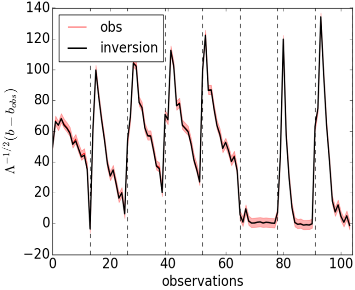

However...

We need more observations!

Ingredients for Great Inversion Result:

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- Choose something the people demand to know ✓

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- Sufficient Number of reliable observations ✓

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- Good Kernels ✓

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- Noise Covariance Matrix ✓

Ingredients for Great Inversion Result:

- Choose something the people demand to know ✓
- Sufficient Number of reliable observations ✓
- Good Kernels ✓
- Noise Covariance Matrix ✓
- Inversion Methods ✓