Equatorial Rossby waves in the solar tachocline

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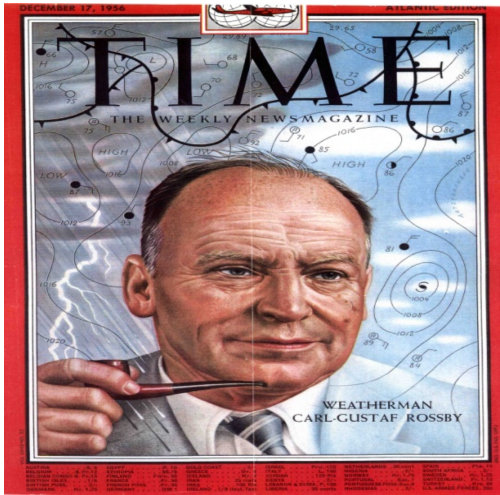
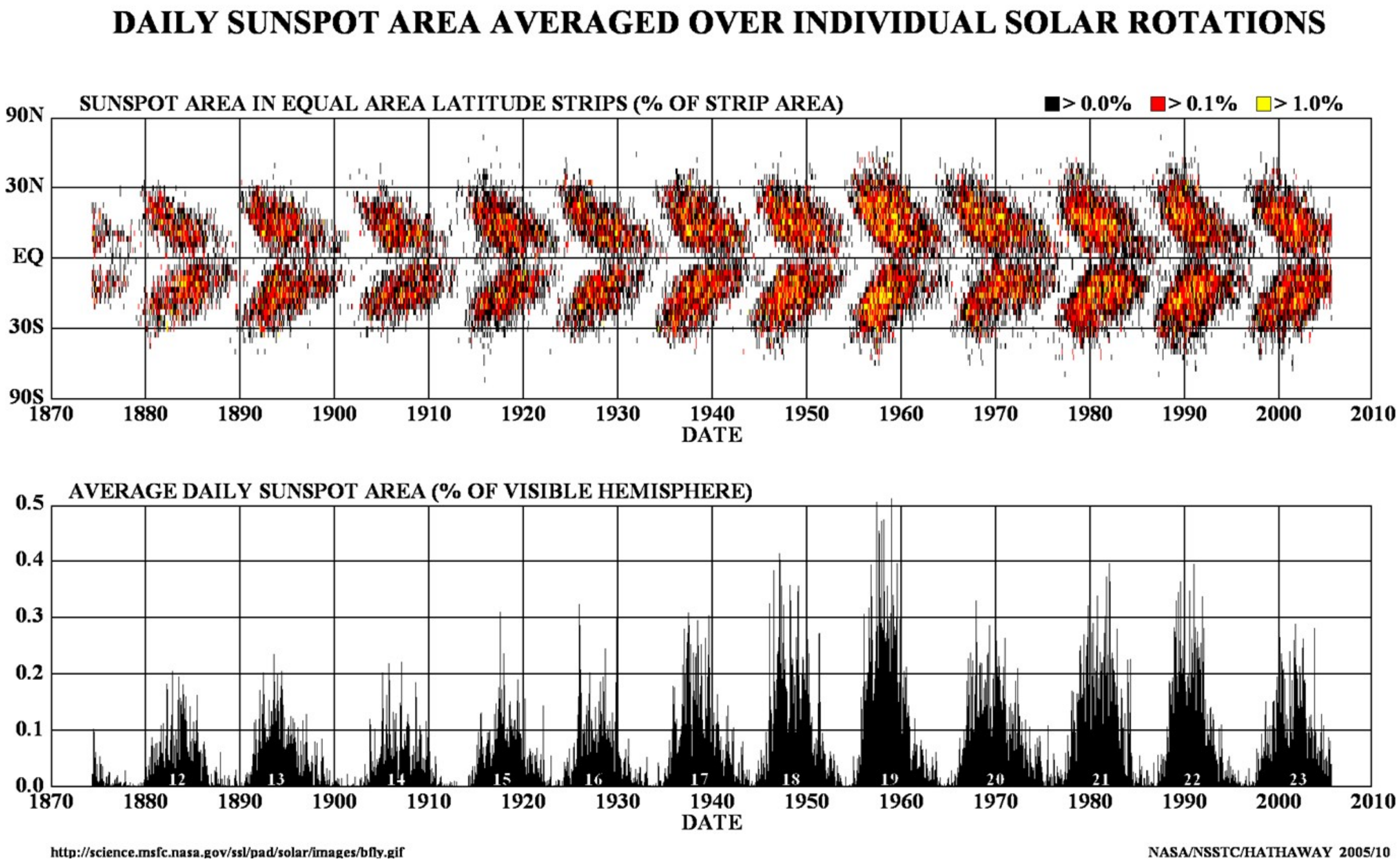
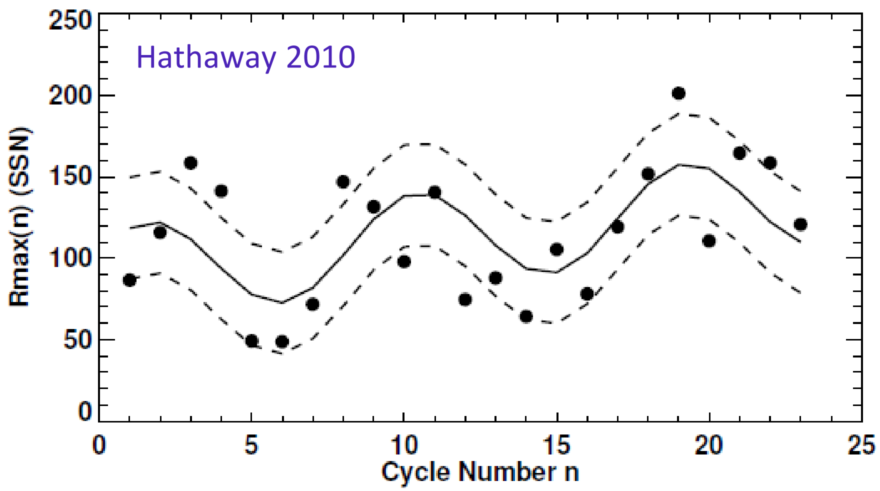
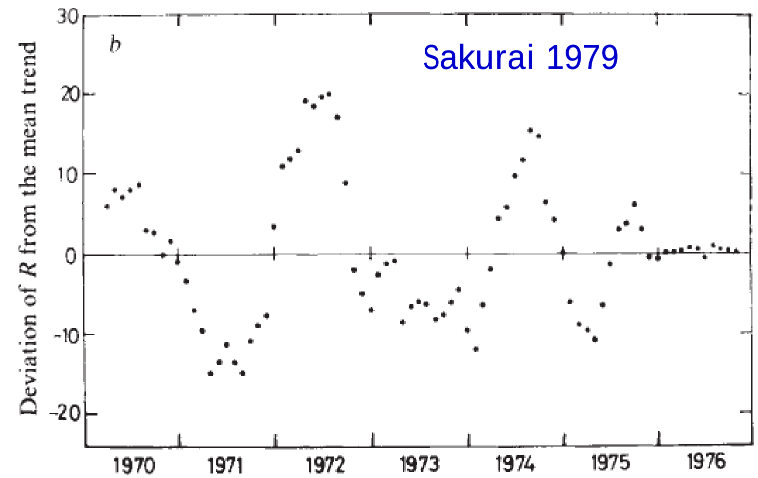
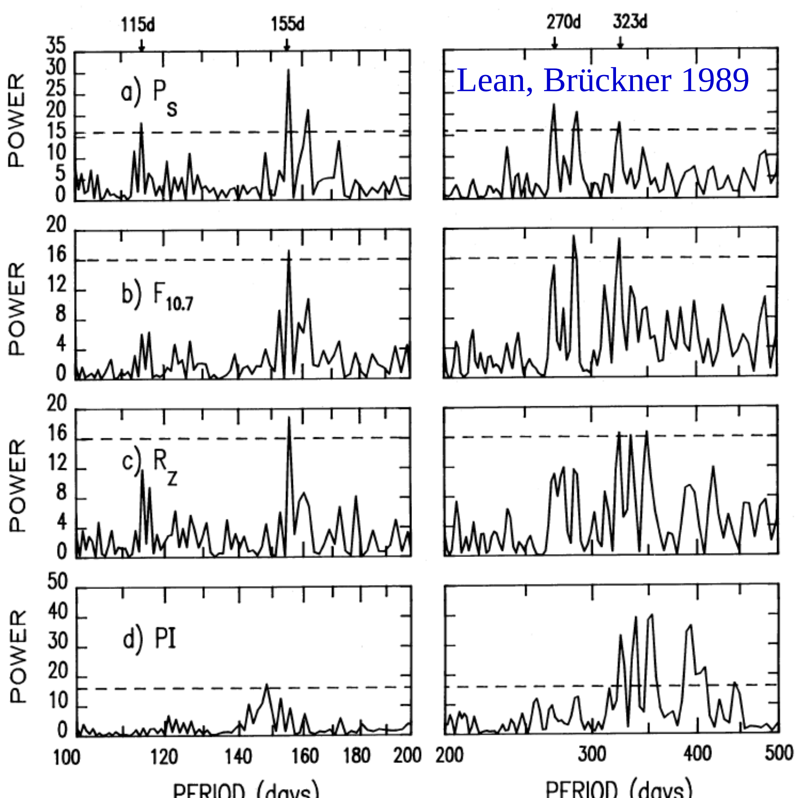




Solar activity undergoes variations over shorter and longer time scales than the period of solar cycles.





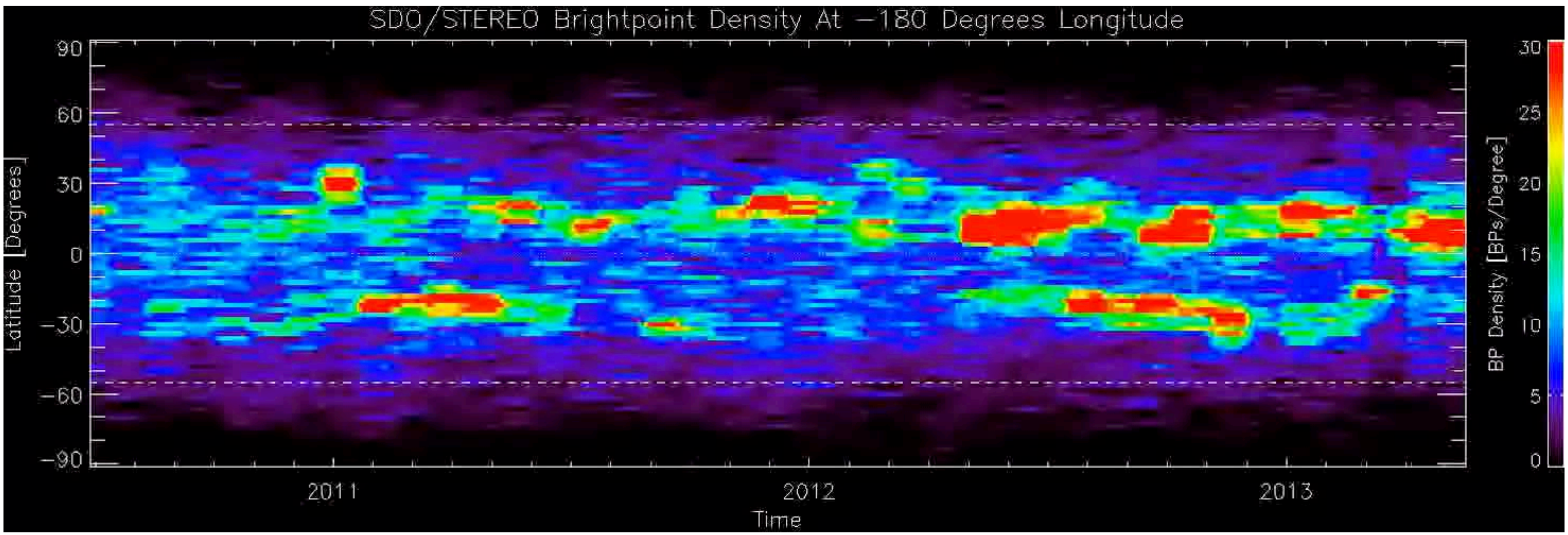


1. Schwabe cycle: 11 yrs
2. Gleissberg cycle: >100 yrs
3. QBO: 1-2 yrs
4. Rieger-type: 150-180 days

Schwabe cycles are usually explained by solar dynamo theory.

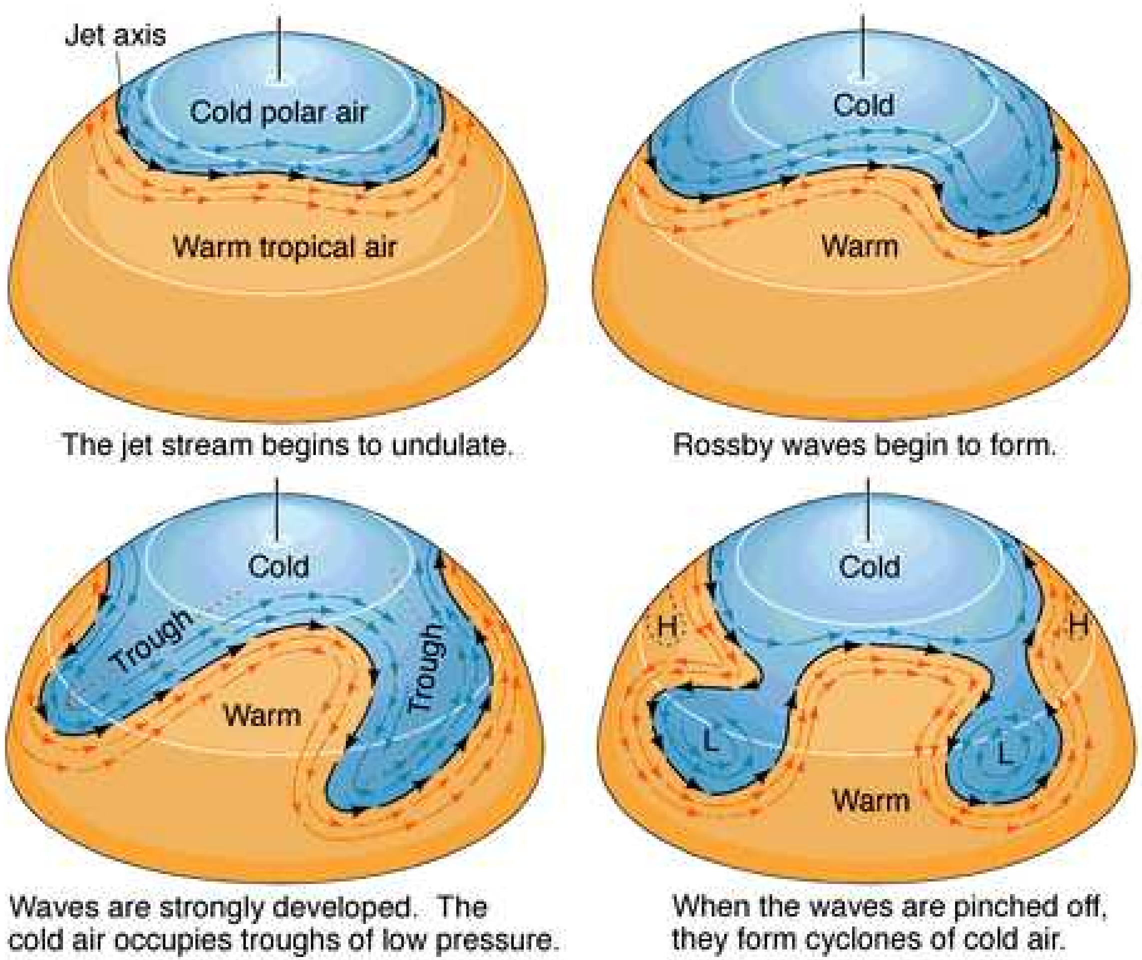
The dynamo is supposed to be operated either in the tachocline (the thin layer below the convection zone) or throughout convection zone (Charbonneau 2005). There is no common agreement to this point today.

Other periodicities are rather purely explained.



McIntosh et al. 2017

Large-scale dynamics of the Earth's atmosphere and oceans are governed by Rossby waves.



MHD linear shallow water equations in the tachocline (Gilman 2000):

$$\frac{\partial u_x}{\partial t} - f u_y = -g\frac{\partial h}{\partial x} + \frac{B_x}{4\pi\rho}\frac{\partial b_x}{\partial x} + \frac{b_y}{4\pi\rho}\frac{\partial B_x}{\partial y}$$

$$\frac{\partial u_y}{\partial t} - f u_x = -g\frac{\partial h}{\partial y} + \frac{B_x}{4\pi\rho}\frac{\partial b_y}{\partial x}$$

$$\frac{\partial b_x}{\partial t} + u_x\frac{\partial B_x}{\partial y} = B_x\frac{\partial u_x}{\partial x}$$

$$\frac{\partial b_y}{\partial t} = B_x\frac{\partial u_y}{\partial x}$$

$$\frac{\partial h}{\partial t} + H\frac{\partial u_x}{\partial x} + H\frac{\partial u_y}{\partial y} = 0$$

$f = 2\Omega\sin\theta$ is the Coriolis parameter.

Reduced gravity is an essential part of shallow water system in the tachocline. It is related with the sub-adiabatic temperature gradient providing a negative buoyancy force to the deformed upper surface (Gilman 2000).

Near the equator, Rossby waves are governed by the equation (Matsuno 1966, Lou 2000):

$$\frac{d^2 u_y}{dy^2} + \left[\frac{\omega^2}{c^2} - k_x^2 + \frac{k_x \beta}{\omega} - \frac{\beta^2}{c^2} y^2\right] u_y = 0 \qquad \beta = \frac{2\Omega}{R} \qquad c = \sqrt{gH}$$

This is the equation of quantum harmonic oscillator and it has bounded solutions, which are trapped around the equator.

The solutions are oscillatory inside the interval $y < \sqrt{\frac{c}{\beta}(2n+1)}$ and exponentially decay outside. Therefore, they are called equatorially Trapped or equatorial waves.

The dispersion relation of the waves is

$$\omega^3 - \left[k_x^2 c^2 + \beta c(2n+1)\right]\omega + k_x \beta c^2 = 0$$

Reduced gravity or negative buoyancy force is proportional to the fractional difference between actual and adiabatic temperature gradients (Dikpati and Gilman 2001), which is in the range of

$|\nabla - \nabla_{ad}| \sim 10^{-4} - 10^{-6}$ in the upper overshoot part of the tachocline

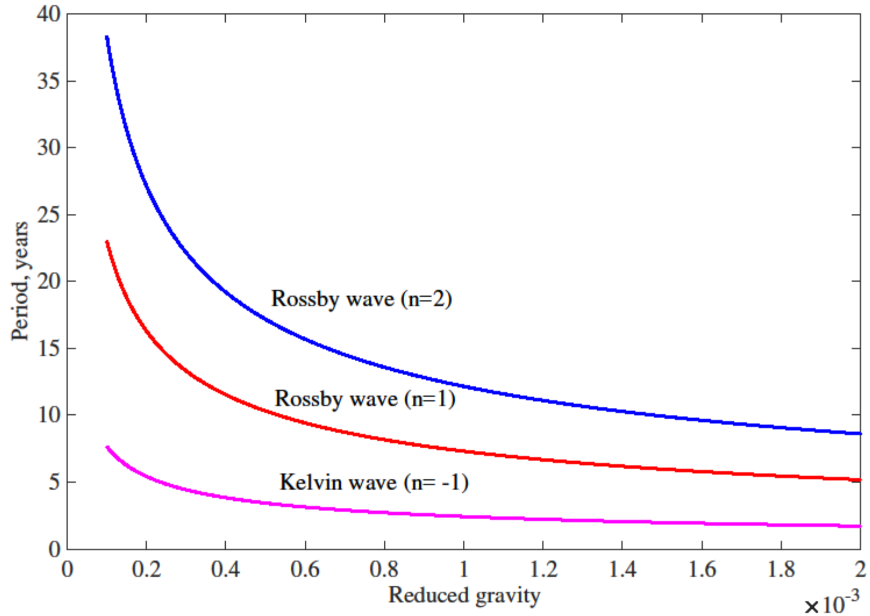
$|\nabla - \nabla_{ad}| \sim 10^{-1}$ in the lower radiative part of the tachocline

Dikpati and Gilman (2001) showed that the dimensionless value of reduced gravity

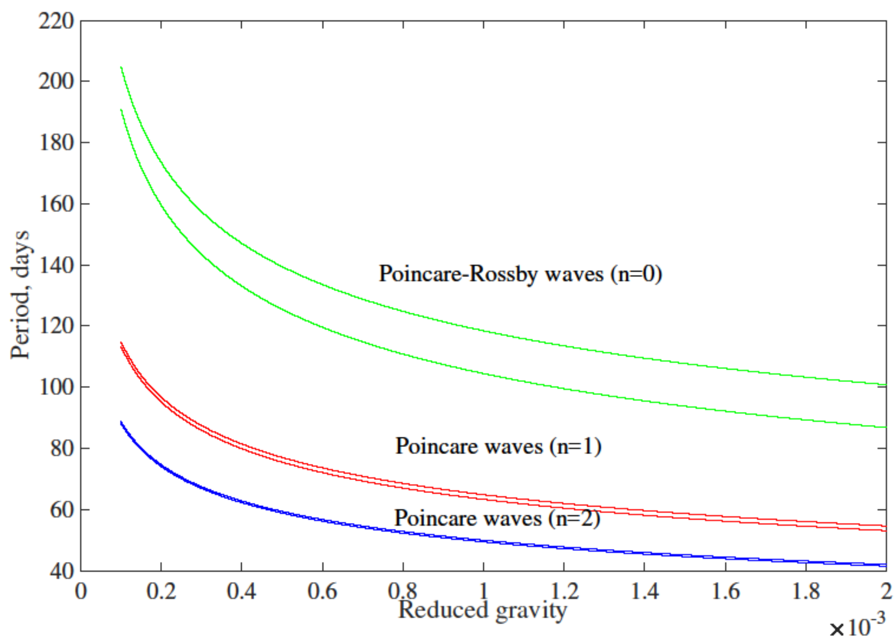
$$G = \frac{gH}{R^2\Omega^2} \sim 10^3 |\nabla - \nabla_{ad}|$$

$10^{-3} \leq G \leq 10^{-1}$ in the upper overshoot

$G > 100$ in the lower radiative



Period, years
Rossby wave (n=2)
Rossby wave (n=1)
Kelvin wave (n= -1)
Reduced gravity
$\times 10^{-3}$



Period, days
Poincare-Rossby waves (n=0)
Poincare waves (n=1)
Poincare waves (n=2)
Reduced gravity
$\times 10^{-3}$

$$\frac{\partial B_\varphi}{\partial t} \sim B_\theta \frac{\partial \Omega_d}{\partial \theta} \qquad \Omega_d = \Omega\left(1 - s_2 \cos^2\theta - s_4 \cos^4\theta\right)$$

Then $B_\varphi \sim B_\theta \left(2\Omega t_0\right) s_2 \cos\theta \sin\theta$

Near the equator $B_x = B_0 \dfrac{y}{R}$

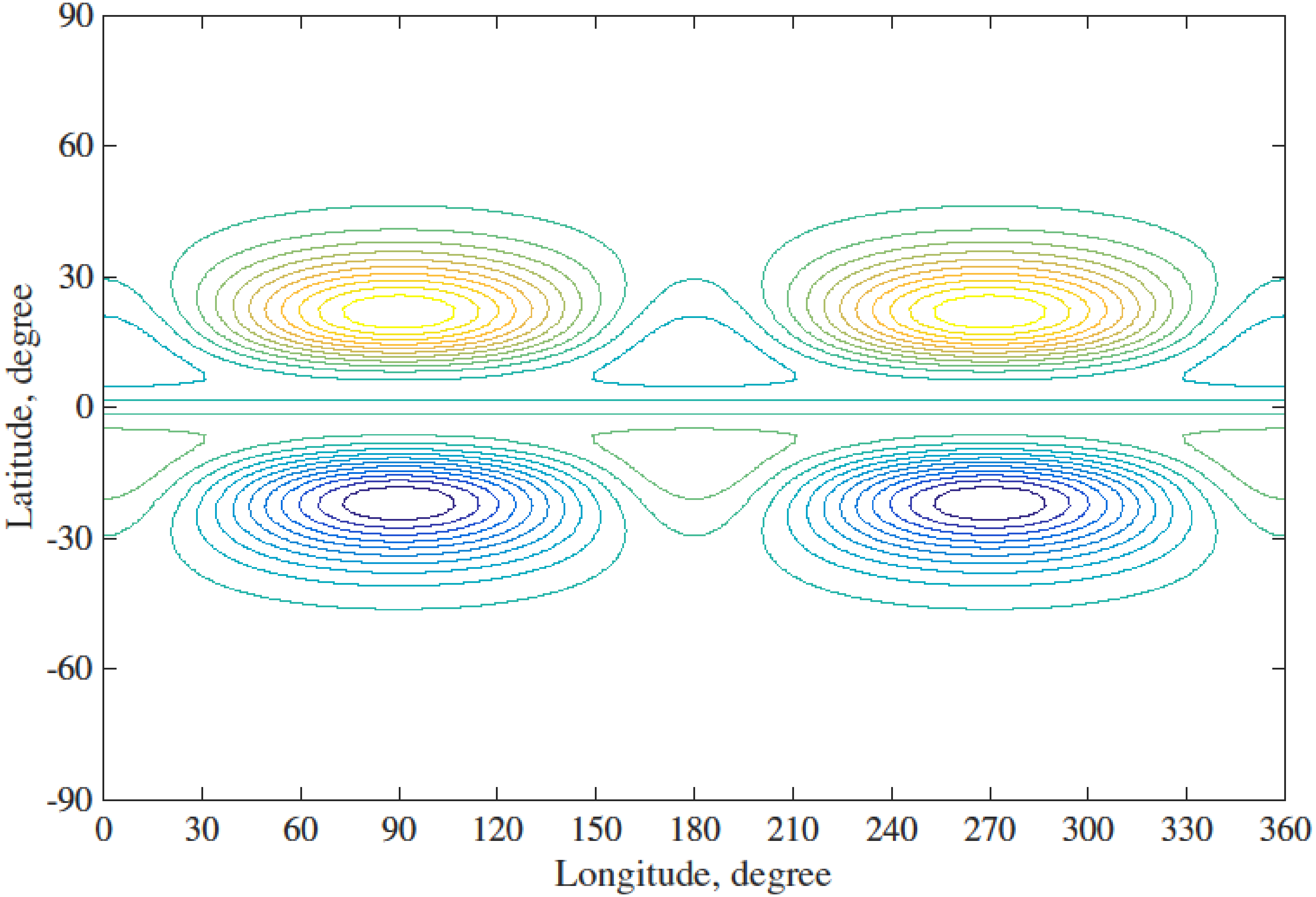
Then the governing equation is

$$\frac{d^2 u_y}{dy^2} + \left[\frac{\omega^2}{c^2} - k_x^2 + \frac{k_x \beta}{\omega} - \frac{k_x^4 c^2 v_A^2}{R^2 \omega^2 \left(\omega^2 - c^2 k_x^2\right)} - \tilde{\mu}^2 y^2\right] u_y = 0$$

Solutions are oscillatory inside the interval

$$y < \sqrt{\frac{2n+1}{|\tilde{\mu}|}}$$

Contour plot of toroidal magnetic field component for n = 2 harmonic.



General dispersion relation is

$$\frac{\omega^2}{c^2} - k_x^2 + \frac{k_x \beta}{\omega} - \frac{k_x^4 c^2 v_A^2}{R^2 \omega^2 \left(\omega^2 - c^2 k_x^2\right)} = (2n+1)\left|\tilde{\mu}\right|$$

Magneto-Poincare waves

$$\omega^3 - \left(k_x^2 c^2 + (2n+1) c \sqrt{\frac{k_x^2 v_A^2 c^2}{R^2} + \beta^2} \right) \omega + k_x \beta c^2 = 0$$

Magneto-Kelvin waves

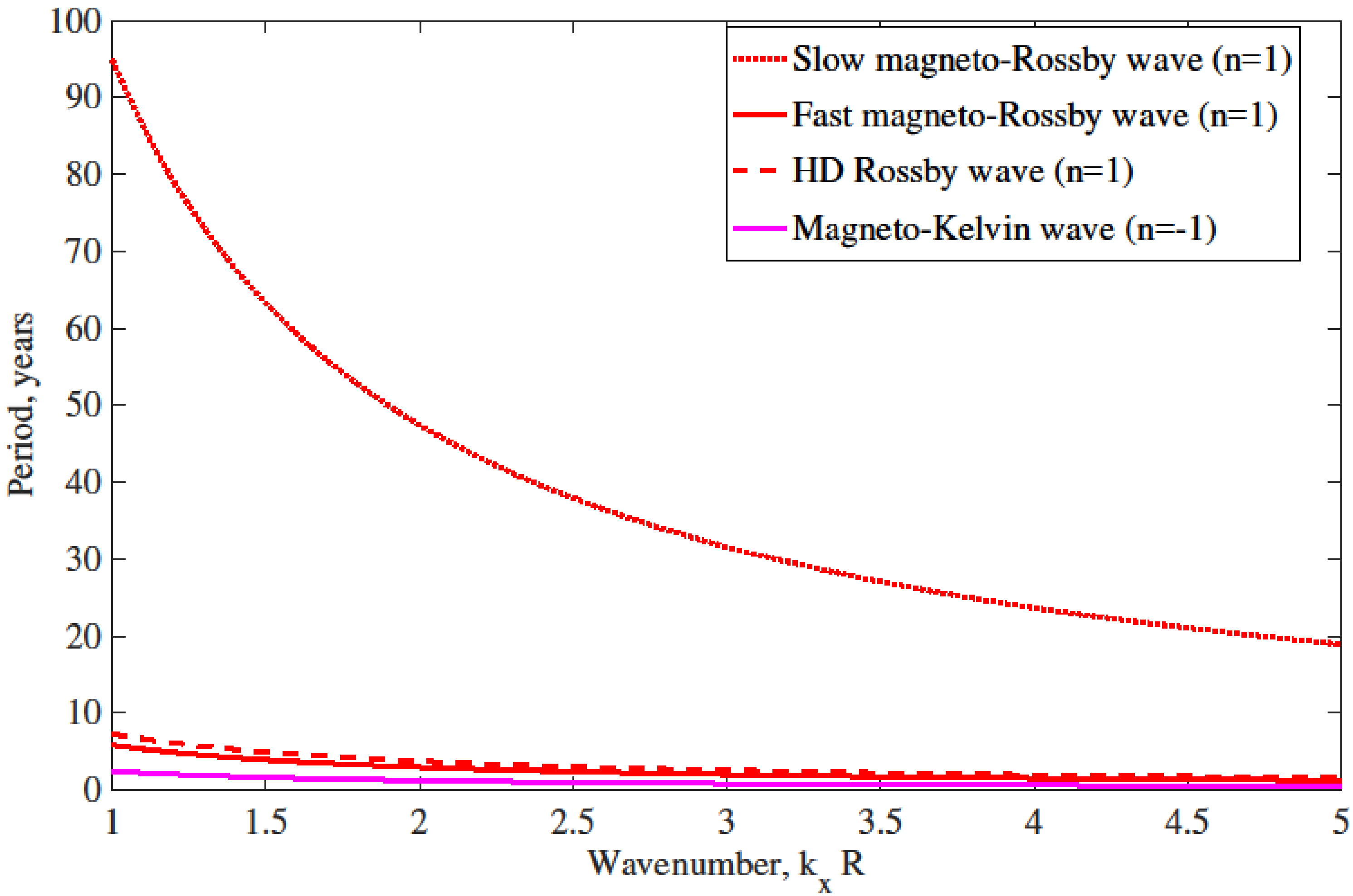
$$\omega = -k_x c$$

Fast magneto-Rossby waves

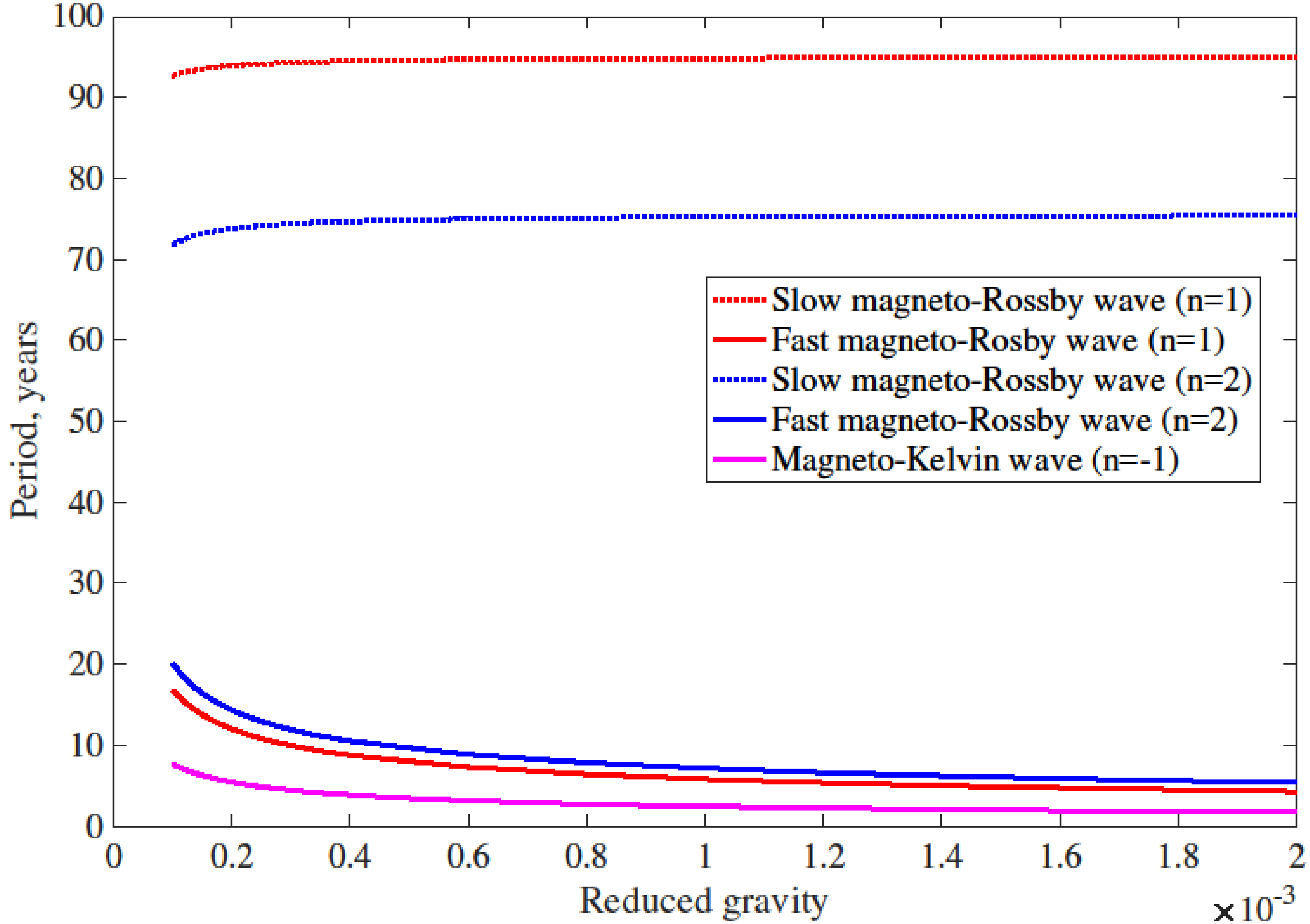
$$\omega \approx \frac{k_x c}{2n+1}$$

Slow magneto-Rossby waves

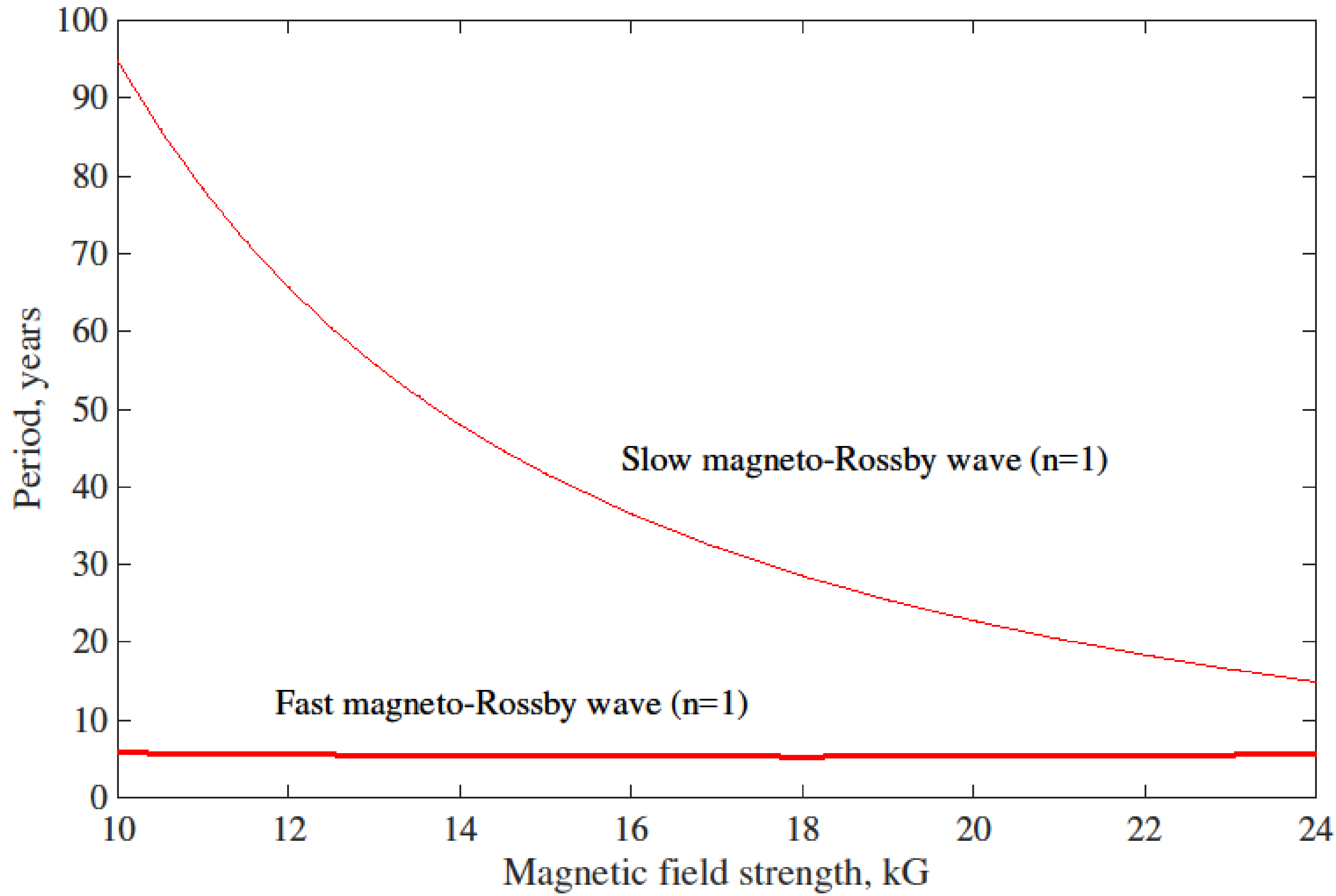
$$\omega \approx \frac{4n}{2n+3} \frac{k_x v_A^2}{\beta R^2}$$



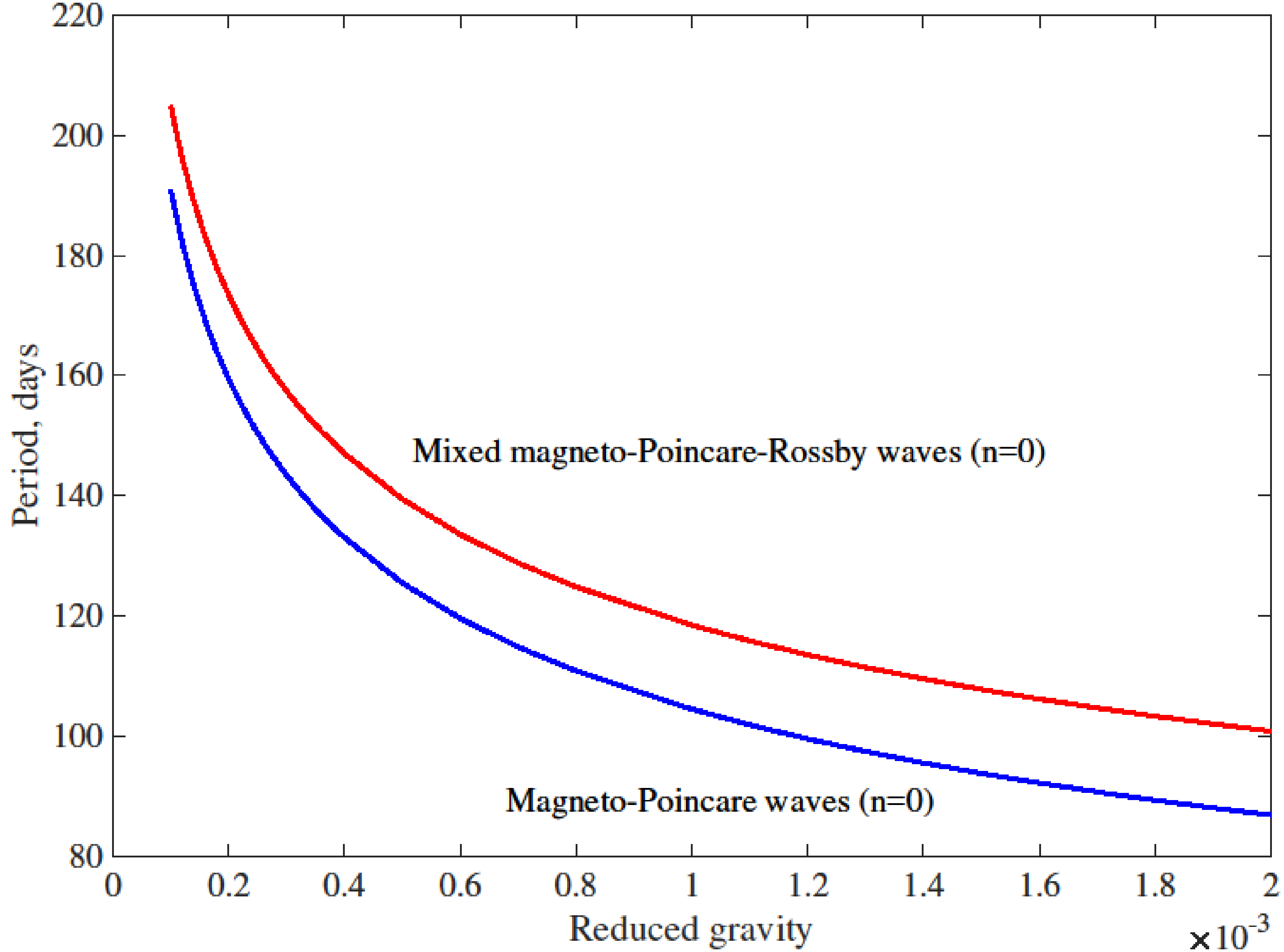
Slow magneto-Rossby wave (n=1)
Fast magneto-Rossby wave (n=1)
HD Rossby wave (n=1)
Magneto-Kelvin wave (n=-1)
Period, years
Wavenumber, k_x R
0
10
20
30
40
50
60
70
80
90
100
1
1.5
2
2.5
3
3.5
4
4.5
5



100
90
80
70
60
50
40
30
20
10
0
Period, years
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Reduced gravity
×10^-3
Slow magneto-Rossby wave (n=1)
Fast magneto-Rosby wave (n=1)
Slow magneto-Rossby wave (n=2)
Fast magneto-Rossby wave (n=2)
Magneto-Kelvin wave (n=-1)



Slow magneto-Rossby wave (n=1)
Fast magneto-Rossby wave (n=1)
Period, years
Magnetic field strength, kG
0
10
20
30
40
50
60
70
80
90
100
10
12
14
16
18
20
22
24



220
200
180
160
140
120
100
80
Period, days
Mixed magneto-Poincare-Rossby waves (n=0)
Magneto-Poincare waves (n=0)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Reduced gravity
×10-3

Conclusions

➢ Shallow water MHD waves are trapped between $\pm 35-40$ latitudes in the upper overshoot tachocline.

➢ Global fast magneto-Rossby waves have period of 11 yrs (Schwabe cycles?)

➢ Global slow magneto-Rossby waves have the period of >100 yrs (Gleissberg cycles?)

➢ Global magneto-Kelvin waves have the period of 1-2 yrs (QBO?)

➢ Global magneto-Poincare waves have the period of 100-200 days (Rieger-type periodicity?)

➢ Detailed analytical/numerical studies are necessary to make conclusion towards the connection of the shallow water waves to the solar activity and hence to the solar dynamo.