Global MHD Tachocline Instabilities

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The Solar Tachocline

- There exists a thin layer (<0.04R), called “tachocline”, which straddles convection zone base (0.7R)
- Tachocline contains strong radial differential rotation along with latitudinal differential rotation at the top that declines to solid rotation at the bottom of tachocline

References:
Brown et al. 1989
Goode et al. 1991
Tomczyk, Chou & Thompson 1995
Kosovichev 1996
Basu 1997
Charbonneau et al. 1997
Corbard et al. 1998
Global HD/MHD Instabilities in Solar Tachocline

Extensive studies in 2D and quasi-3D Shallow-water and thin-shell type models of solar tachocline indicate the existence of global HD/MHD instabilities in tachocline with low longitudinal wave numbers

(Arlt, Cally, Charbonneau, Dikpati, Dziembowski, Garaud, Gilman, Kosovichev, Miesch, Ruediger, Zaqarashvili)
2D HD Tachocline

\[ \omega_0 = s_0 - s_2 \mu^2 \]

\[ \mu = \sin () \]

\[ \omega_0 = s_0 - s_2 \mu^2 - s_4 \mu^4 \]

This differential rotation profile was found to be hydrodynamically stable for the Sun (Watson 1981)

Hydrodynamically unstable for the Sun (Dziembowski and Kosovichev, 1987; Charbonneau, Dikpati and Gilman 1999)

\[ \omega_0 = s_0 - s_2 \mu^2 \]

\[ + \alpha_0 = a\mu + b\mu^3; \text{ node at } \mu = \left(-\frac{a}{b}\right)^{1/2} \]

Magnetohydrodynamically unstable (Gilman and Fox 1997; Dikpati and Gilman 1999; Cally 2001; Zaqarashvili et al. 2009)
Properties of 2D MHD Instability of Differential Rotation and Toroidal Magnetic Field

Angular momentum transport toward the poles primarily by the Maxwell Stress (perturbations field lines tilt upstream away from equator)

Magnetic flux get transported away from the peak toroidal field by the Mixed Stress (phase difference in longitude between perturbation velocities & magnetic fields)
Schematic of Possible Modes of Instability in 2D MHD Tachocline

- Toroidal ring shrinks
- Fluid in ring spins up

- Toroidal ring deforms, creating Maxwell Stress
- Fluid inside ring deforms but does not spin up

- Toroidal ring tips but remains same circumference
- Fluid in ring keeps same speed
Clamshell Instability *(Cally 2001)*

Can the poloidal magnetic fields generated be sheared to produce toroidal magnetic fields again, and how fast?
Nonlinear Tipping of Toroidal Fields

Cally, Dikpati and Gilman, 2003; Dikpati, Cally and Gilman, 2004
Energy Flow Diagram for Nonlinear 2D MHD System

(Dikpati, Cally and Gilman, 2004)
Shallow Water System

- Spherical Shell of fluid with outer boundary that can deform
- Horizontal flow, fields in shell are independent of radius
- Vertical flow, field linear functions of radius, zero at inner boundary
- Magnetohydrostatic force balance
- Horizontal gradient of total pressure is proportional to the horizontal gradient of shell thickness
- Horizontal divergence of magnetic flux in a radial column is zero

Pedlosky, Longuet & Higgins 1970; Gilman, 2000; Dikpati & Gilman 2001; Zaqarashvili 2010; Dikpati 2012,
Equilibrium in Shallow-water HD/MHD Tachocline

In general, is a balance among three latitudinal forces, including hydrostatic pressure gradient, magnetic curvature stress, and coriolis forces.

Dikpati and Gilman 2001
Effective Gravity Parameter ($G$)

$$G = \frac{1}{2} \frac{g_t \left| \nabla - \nabla_{ad} \right| H^2}{(r_t \omega_c)^2 H_p}$$

in which:

- $g_t$: gravity at tachocline depth
- $\left| \nabla - \nabla_{ad} \right|$: fractional departure from adiabatic temperature gradient
- $H$: thickness of tachocline “shell”
- $H_p$: pressure scale height
- $r_t$: solar radius at tachocline depth
- $\omega_c$: rotation of solar interior

$G \sim 10^{-1} - 1$ for Overshoot Tachocline

$G \sim 10 - 10^3$ for Radiative Tachocline
Shallow Water Equations of Motion and Mass Continuity

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -G \frac{1}{\cos \phi} \frac{\partial h}{\partial \lambda} + \frac{v}{\cos \phi} \left[ \frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi} (u \cos \phi) \right] - \frac{1}{\cos \phi} \frac{\partial}{\partial \lambda} \left( \frac{u^2 + v^2}{2} \right) \\
&\quad - \frac{b}{\cos \phi} \left[ \frac{\partial b}{\partial \lambda} - \frac{\partial}{\partial \phi} (a \cos \phi) \right] + \frac{1}{\cos \phi} \frac{\partial}{\partial \lambda} \left( \frac{a^2 + b^2}{2} \right),
\end{align*}
\]

\[
\begin{align*}
\frac{\partial v}{\partial t} &= -G \frac{\partial h}{\partial \phi} - \frac{u}{\cos \phi} \left[ \frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi} (u \cos \phi) \right] - \frac{\partial}{\partial \phi} \left( \frac{u^2 + v^2}{2} \right) \\
&\quad + \frac{a}{\cos \phi} \left[ \frac{\partial b}{\partial \lambda} - \frac{\partial}{\partial \phi} (a \cos \phi) \right] + \frac{\partial}{\partial \phi} \left( \frac{a^2 + b^2}{2} \right),
\end{align*}
\]

\[
\begin{align*}
w &= \frac{\partial}{\partial t} (1+h) = -\frac{1}{\cos \phi} \frac{\partial}{\partial \lambda} [(1+h)u] - \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} [(1+h) v \cos \phi],
\end{align*}
\]

Pedlosky, Longuet & Higgins 1970;
Shallow Water Induction and Flux Continuity Equations

\[
\frac{\partial a}{\partial t} = \frac{\partial}{\partial \phi} (ub - va) + \frac{a}{\cos \phi} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos \phi) \right] - \frac{u}{\cos \phi} \left[ \frac{\partial a}{\partial \lambda} + \frac{\partial}{\partial \phi} (b \cos \phi) \right],
\]

\[
\frac{\partial b}{\partial t} = -\frac{1}{\cos \phi} \frac{\partial}{\partial \lambda} (ub - va) + \frac{b}{\cos \phi} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos \phi) \right] - \frac{v}{\cos \phi} \left[ \frac{\partial a}{\partial \lambda} + \frac{\partial}{\partial \phi} (b \cos \phi) \right],
\]

\[
\frac{1}{\cos \phi} \frac{\partial}{\partial \lambda} [(1 + h)a] + \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} [(1 + h)b \cos \phi] = 0.
\]

(Gilman & Dikpati 2002)
Singular Points

Occur at latitudes where:

\[ S_r = \omega_o - c_r = 0 \]
\[ S_m = (\omega_o - c_r)^2 - \alpha_o^2 = 0 \]
\[ S_g = (1 - \mu^2) \left[ (\omega_o - c_r)^2 - \alpha_o^2 \right] - G(1 + h_o) \]

\( h_\sigma \) is departure of shell thickness from uniform thickness

For cases of solar interest:

\( S_r, S_m = 0 \) are important, \( S_g = 0 \) is not

- Singular points define places of rapid phase shifts with latitude in unstable modes
- Therefore much of disturbance structure, as well as energy conversion processes, can occur in this neighborhood
- Singular points play major role in interpreting how this instability is driven
Stability Diagrams for HD Shallow Water System

\[ |\nabla - \nabla_{ad} | \]

(Dikpati and Gilman, 2001)
Solving Nonlinear Shallow-water Equations in Spherical Coordinate

- Start with real space first order shallow-water equations
- Decompose $h$, $u$ and $v$ in scalar and vector spherical harmonics to deal with the pole problem
- Evaluate nonlinear terms by implementing pseudo-spectral scheme (Swarztrauber 1996)
- Implement semi-implicit scheme to take the advantage of larger time-step so that high-frequency gravity waves are integrated out (Hack & Jacob 1992)
- Implement fourth order Runge-Kutta time integration scheme
- Parallelize forward and backward transforms in shared memory parallelization scheme (Dikpati 2012)
Nonlinear evolution of shallow-water instability

Arrow vectors: Global flow (clockwise in swelling, and anticlockwise in depression)
Color shades: Tachocline thickness (red represents swelling, blue depression)
Flow is nearly geostrophic in a hydrodynamic or weakly magnetized tachocline

Features to note in simulation:

• Quasi-periodic oscillations in thickness and velocities
• Slow retrograde propagation relative to rotating frame
• Flow nearly geostrophic in hydrodynamic tachocline, and magnetostrophic in magnetized tachocline
• Occasional appearance of gravity waves
Nonlinear evolution of shallow-water instability

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Evolution of kinetic and potential energies

- Interaction between reference state and perturbation kinetic energies is oscillatory.
- These oscillations are related to oscillatory interaction between Rossby waves and differential rotation.
- These periodicities organize space weather that influences the earth.

(Dikpati 2012)
Solar “seasons”, Rieger-type periodicity, QBO

McIntosh et al. 2015; Rieger et al. 1984; Oliver et al. 1998; Ballester et al. 1999; Krivova & Solanki 2002; Zaqarashvili et al. 2010; Gurgenashvili et al. 2016, 2017
What causes the quasi-periodic solar bursts?

Zaqarashvili et al. (2010) showed that the interactions of oscillatory neutral modes and growing modes in an MHD shallow water model can explain the Rieger-type periodicities and QBO.

We recently showed (Dikpati et al. 2017) that the Tachocline Nonlinear Oscillations (TNO), occurring between Rossby waves and differential rotation can cause enhanced activity bursts.

Recent study (Gurgenashvili et al. 2017) showed that quasi-periodicity has North-South asymmetry, and dependence on magnetic field strength.
Schematic of interaction between differential rotation and Rossby waves

-ve momentum going towards equator

+ve momentum going away from equator

N pole

Equator

S pole

Resulting pattern (solid curves)

no net transport poleward or equatorward

N

Eq

S

+ve momentum going towards equator

-ve momentum going away from equator

N

Eq

S

HAO

Schematic of interaction between differential rotation and Rossby waves
Causes of solar “seasons”

Dikpati, Cally, McIntosh & Heifetz 2017, submitted
Periodicity of solar seasons as function of model parameters

(a) Pole-to-equator DR amplitude (%)

(b) Effective gravity G

(c) Field strength

(d) Latitude location of peak field

TNO Period (months)

Amplitude of magnetic band (kGauss)

Latitude of magnetic band (degree)
High latitude jets

Reynolds stress transports angular momentum towards the poles

(Dikpati, 2012)
MHD Shallow-water Instability: A Theory of Active Longitudes

Portion of toroidal band that starts entering convection zone and making its buoyant rise
Initial toroidal field is the unperturbed toroidal band with 20 kG peak field plus perturbations caused by all plausible unstable longitudinal modes (m=1S, m=1A, m=2A here)

Identify longitude locations of the band that coincide with the swelled fluid

Active longitudes evolve according to the propagation of unstable MHD shallow-water modes

Evolution of theoretical and observed active longitudes

Dikpati and Gilman, 2005
Evolution of swelling/depression of tachocline fluid and observed active longitudes

(Dikpati and Gilman, 2005)
Consequences of 3D MHD axisymmetric \((m=0)\) instability with radial wavenumber 5

Toroidal band deforms more in its poleward side

Flow extends further in the latitude direction than the magnetic band

It seems that flows are carrying the field away from or toward the maximum field, but in fact opposite is happening; magnetic curvature forces are pushing the fluid along with the moving field

Fields fragment in radial direction, narrower bands form and get stacked in depth

Dikpati, Gilman, Cally & Miesch 2009
Summary

- HD/MHD 2D as well as shallow-water instabilities can occur in solar tachocline for a wide range of parameters, such as DR, G, toroidal field strengths and latitude location of fields.
- The quasiperiodic solar “Seasons”, or characteristic time variations in solar activity on a scale of 6 – 18 months, can be simulated by using MHD Shallow-water tachocline model, and can be forecast 2 years ahead when surface observations are assimilated.
- Rossby waves (Zaqarashvili et al. 2015) as well as principal components of temporal magnetic field variations (Zharkova et al. 2015) over the past few solar cycles can also be used for long-term predictions of solar activity over hundreds and even thousands of years.
- When a certain longitude portion of a toroidal band coincide with the swelled fluid region due to tipping of the band, that portion rises to the surface to produce spots at that longitude, creating “active longitudes”
Thank you
Stability Diagrams for MHD Shallow Water System

Overshoot Layer

Radiative Layer

(Dikpati, Gilman and Rempel, 2003)