The model of solar wind polytropic flow patterns

work in progress

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in collaboration with

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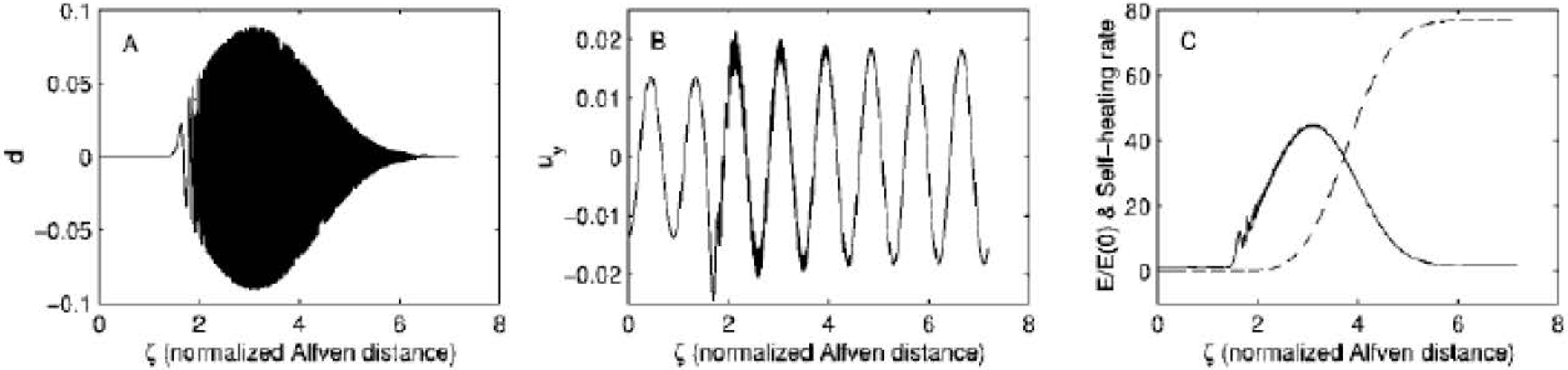
Our Mysterious Sun: magnetic coupling between solar interior and atmosphere

Tbilisi, 25th -29th September 2017

- Introduction: motivation
- Introduction: why patterns? Types of patterns
- Introduction: large-scale polytropic flow patterns – the solar wind
- Mathematical formulation and physical content of the integration constants
- Family of analytic solutions for monatomic plasma with adiabatic index equal to 5/3
- Building the pattern using in-situ, remote-sensing and radio observational data
- Analytic solutions for adiabatic index not equal to 5/3
- Implications for partially ionized plasma
- Conclusions and the planned work

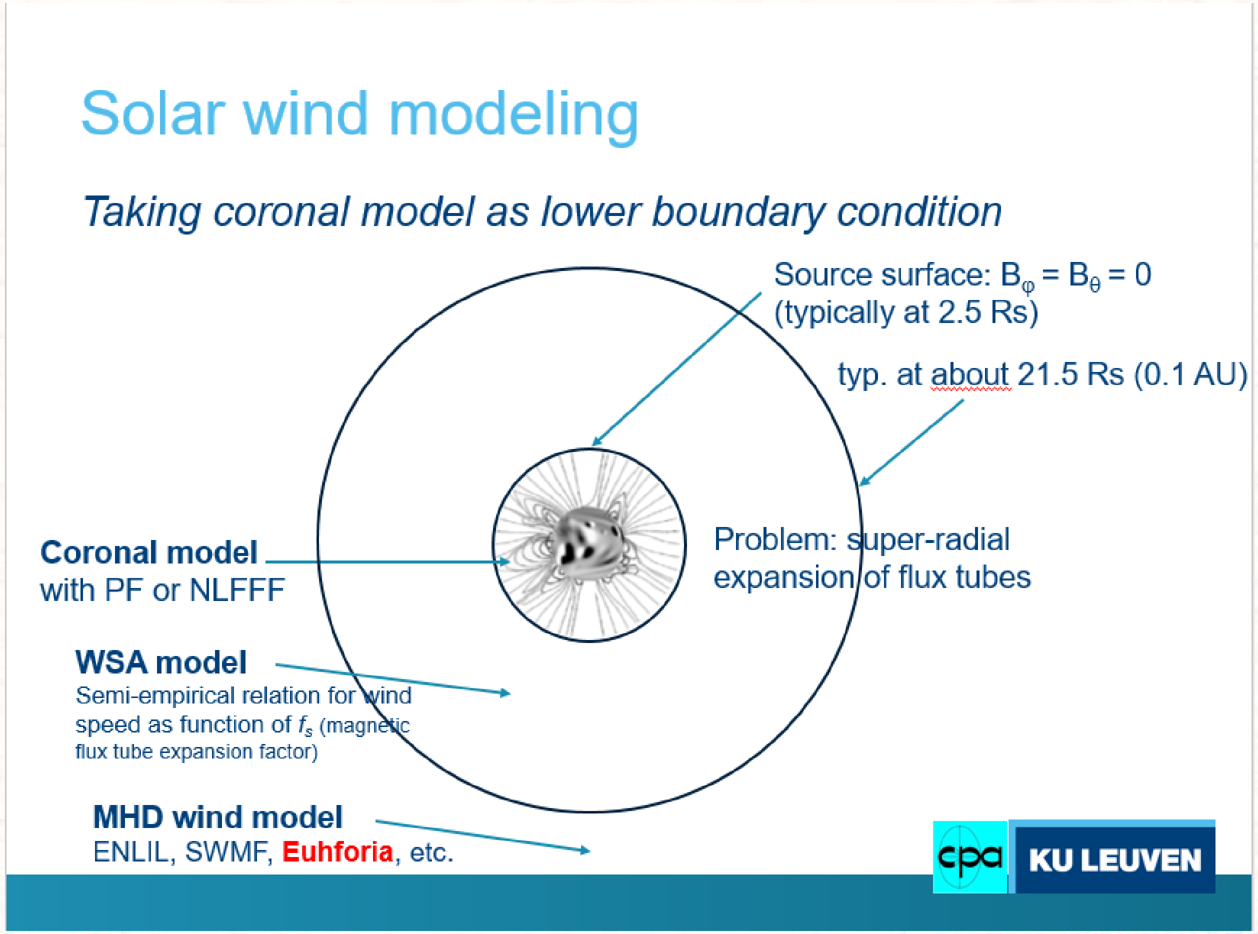
- Need for modelling of MHD wave shear flow driven dynamics in the solar corona and wind



Plots of different physical quantities against the dimensionless time variable $\zeta = v_A t / R_\odot$*, representing the Alfvén distance in units of the solar radius: (a) Normalized density perturbation* d*; (b) dimensionless* y*-component of the velocity perturbation* u_y*; (c) total perturbation energy normalized by its initial value (solid line) and the self-heating function (dashed line).*

From Shergelashvili et al (2006)

- Need for the consistent parametrization of the velocity, density and temperature coronal (source region) profiles for the solar wind numerical models. Space weather and planetary science applications.
- Need for understanding of the Sun and the heliosphere as coupled, unified system in responce to the Solar orbiter and other forthcoming missions.
- Need for the proper clasification of the solar wind flows into the fundamental pattern libraries (ontologies) for the realization of the large observational and numerical dataset processing using methods of artificial intelligence.



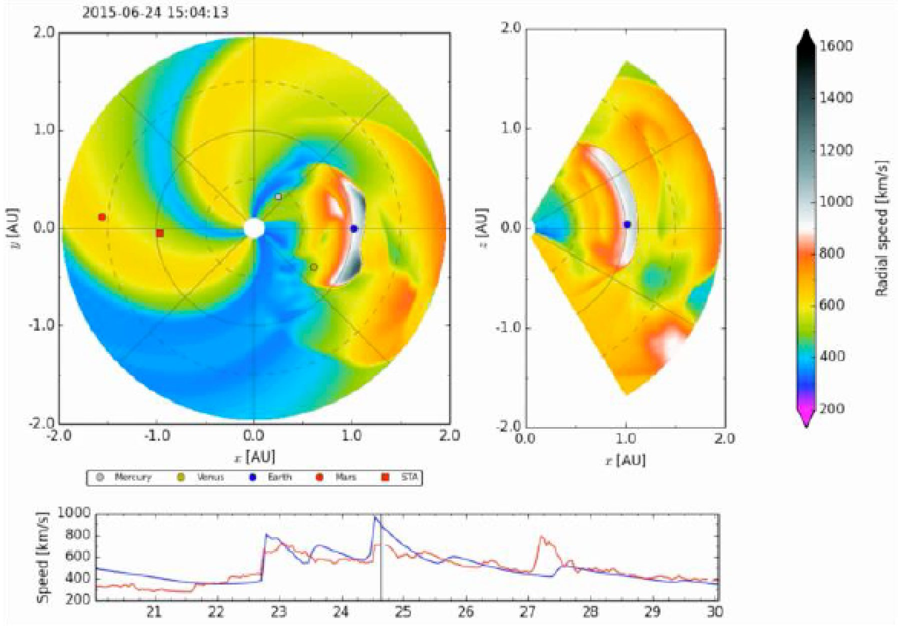
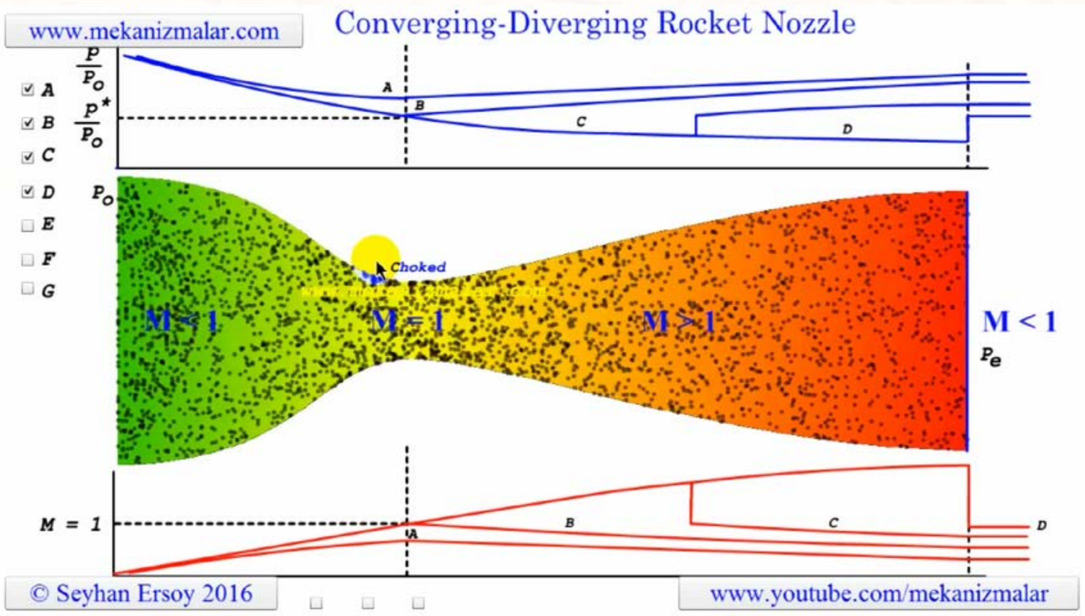


Figure 5. Snapshot of a forecast simulation with EUHFORIA, showing the radial velocity in the equatorial plane (top left, viewed from above) and in the meridional plane through the position of Earth (top right, side view). Bottom: comparison of simulated (EUHFORIA, in blue) and measured (ACE, in red) radial velocity at L1 [from J. Pomoell].

- Eugene Parker was the first who realized that there is a direct analogy between the engineering subsonic/supersonic nozzles (diffusers) and transsonic velocity profiles in the stellar atmospheres embedded in the gravitational field.



Case of α=1, Parker 1965

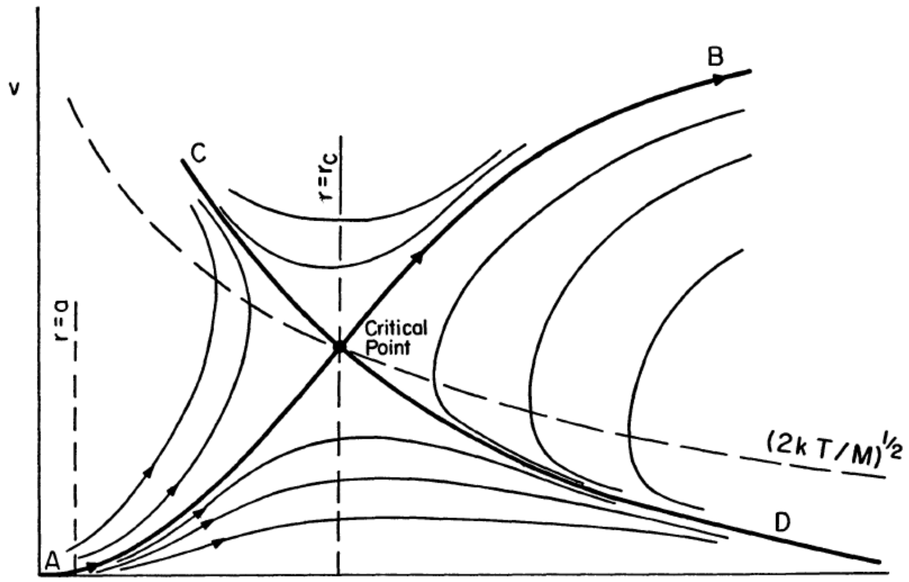


FIG 1 —Sketch of the one-parameter family of solutions of the momentum equation (8) for a temperature declining less rapidly than $1/r$. The solutions have $dv/dr = 0$ at $r = r_c$, and $dv/dr = \infty$ on the curve $(2kT/M)^{1/2}$. The position $r = a$ represents the base of the corona.

$$\frac{r}{v}\frac{dv}{dr} = \frac{2C_s^2 - \frac{GM_\odot}{r}}{v^2 - C_s^2},$$

$$r = r_c = \frac{GM_\odot}{2C_s^2} = R_\odot \frac{V_{\odot esc}^2}{4C_s^2},$$

- We consider the model of the polytropic of the spherical star atmosphere expanding towards the void surrounding it. Multi-fluid version of the model has been developed by Summers (1982).

- We consider the stationary, adiabatic wind flow patterns.

Governing equations

$$r^2 \rho v = Q_\rho = const,$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM_\odot}{r^2},$$

$$\frac{d}{dr}\frac{p}{\rho^\alpha} = 0,$$

Auxiliary function:

$$r_c(r) = \frac{GM_\odot}{2C_s^2(r)}, \qquad \frac{d\ln r_c}{dr} = -\frac{d\ln C_s^2}{dr},$$

System of two differential equations is obtained:

$$(5-3\alpha)\frac{d\ln C_s^2}{dr} + (\alpha-1)\left(\frac{d\ln\xi}{dr} + 4\frac{d\ln\eta}{dr}\right) = 0,$$

$$\frac{d\xi}{dr} - \frac{d\ln\xi\eta^4}{dr} - (3+\xi)\frac{d\ln r_c}{dr} = -\frac{4r_c}{r^2},$$

$\eta = r/r_c$ (dimentionless distance variable) $\xi = M^2$ (Mach number squared)

In case of α=5/3 the system is integrable and analytic solution is available:

$$\xi\eta^4 = C_*^2 = const,$$

$$\frac{d\xi}{dr} - (3+\xi)\frac{d\ln r_c}{dr} = -\frac{4r_c}{r^2},$$

$$\frac{d}{dr}\ln\frac{\eta\left(3 + C_*^2\eta^{-4}\right) - 4}{r} = 0,$$

$$3\eta^4 - (4 + Dr)\,\eta^3 + C_*^2 = 0,$$

$$C_* = \frac{16Q_\rho Q_{Cs}^{3/2}}{R_\odot^2 V_{\odot esc}^4}, \qquad Q_{Cs} = \frac{C_s^2}{\rho^{\frac{2}{3}}} = const,$$

It is convenient to intruduce following notation

$$\Delta = \left(4\Delta_0^3 - \Delta_1^2\right)/27 = 256a^3e^3 - 27b^4e^2 = 27C_*^4\left(256C_*^2 - b^4\right)$$

$$K = 64a^3e - 3b^4 = 3\left(576C_*^2 - b^4\right),$$

here, $a = 3$, $b = -(4 + Dr)$ and $e = C_*^2$, $\Delta_1 = 27b^2e$.

$\Delta_0 = 12ae$

No flow solution: $\Delta = 0,$ $\Delta_0 = 0$ and $K \neq 0$ $C_* = 0$ $Q_\rho = r^2 \rho v = 0.$

In this case equation has one physically meaningless triple solution $\eta = 0$ and one non trivial solution:

$$\eta = \frac{4 + Dr}{3}, \qquad C_s^2 = Q_{Cs} \rho^{\frac{2}{3}} = \frac{GM_\odot D}{6}\left(1 + \frac{4}{Dr}\right)$$

Existence of the critical point with M=1 enabling transonic solution patterns implies that above the solar surface exists $r = r_*$ where the equation has double solution. $\Delta = 0,$ $K > 0$

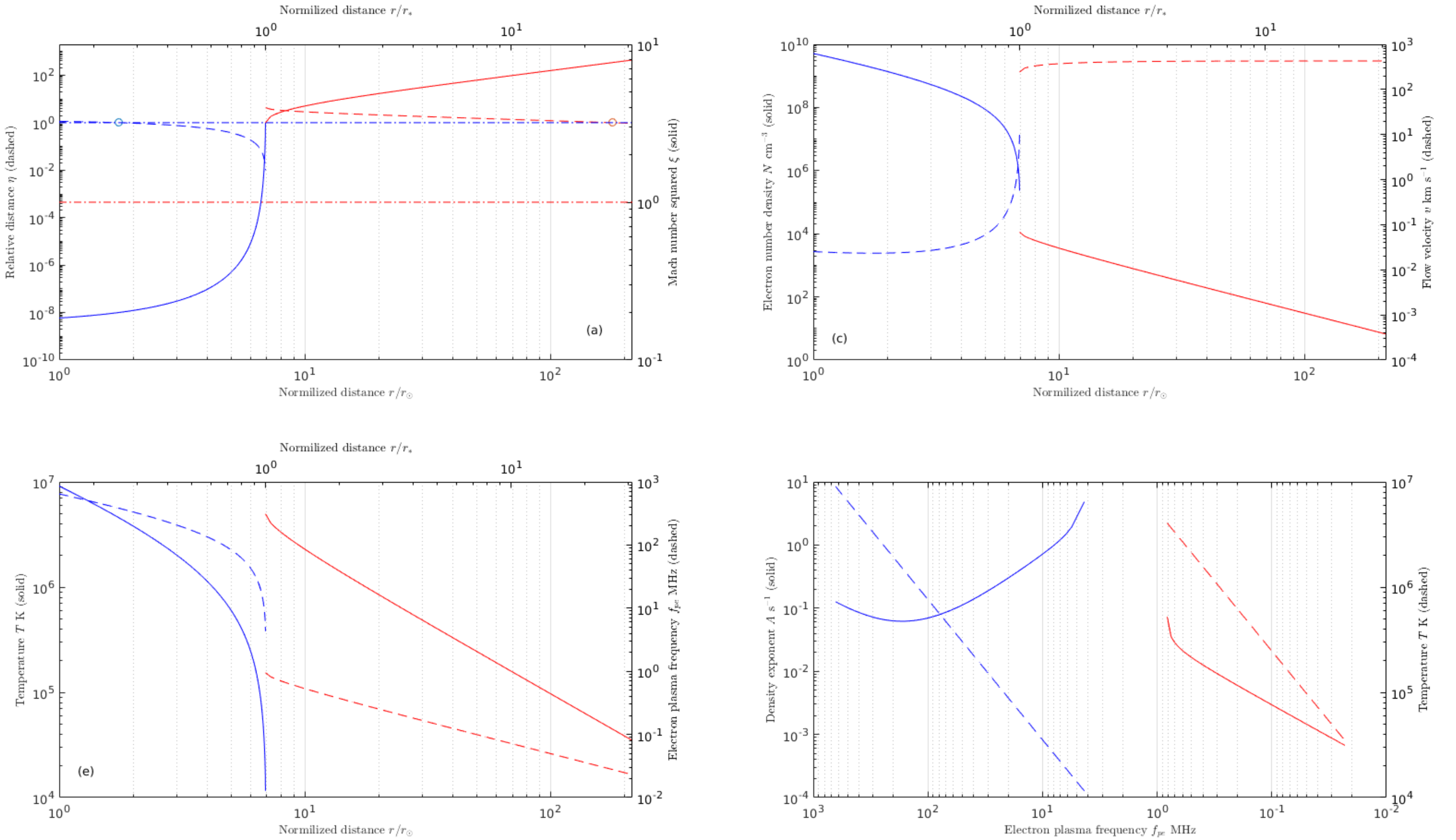
$b = -4\sqrt{C_*}$ $\eta_* = \sqrt{C_*}$ is the unique double solution of $3\eta^4 - 4\sqrt{C_*}\eta^3 + C_*^2 = 0.$

One can construct the final form of the equation having transsonic pair of solutions

$$3\eta^4 - 4\left(1 + \left(\sqrt{C_*} - 1\right)\frac{r}{r_*}\right)\eta^3 + C_*^2 = 0.$$

$$r_* = \frac{4\left(\sqrt{C_*} - 1\right)}{D},$$

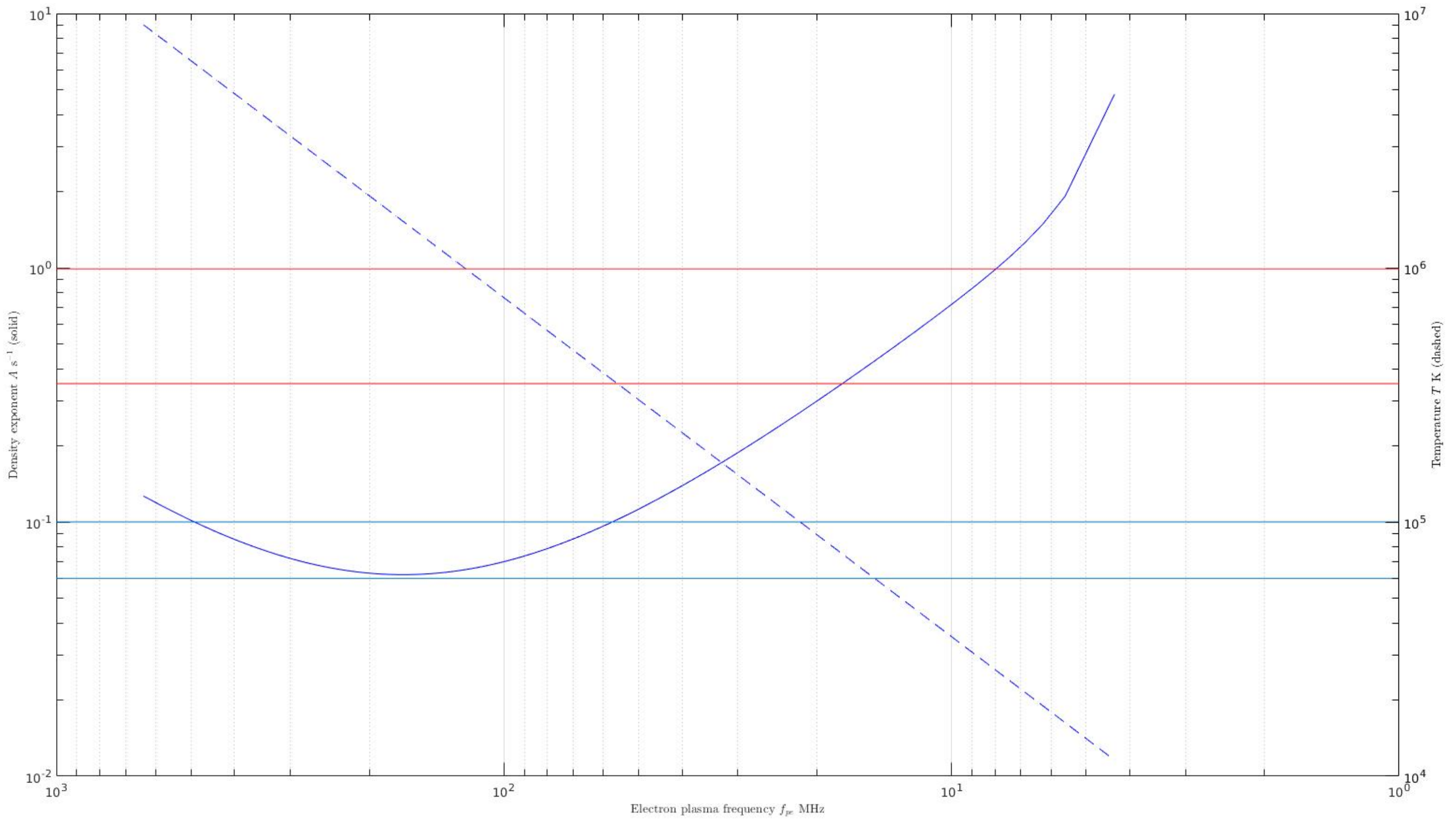
We present sample solution corresponding to the slow wind pattern. Parameters are taken from Mann et al. (1999)



The critical point is at 6.91 solar radii, the velocity at 1AU is 427 km/s

The shape of the density profiles in the corona and beyond is linked with the solar radio observational data

$$\frac{df}{dt} = f \cdot \frac{1}{2n_e} \cdot \frac{dn_e}{dr} \cdot v_s,$$

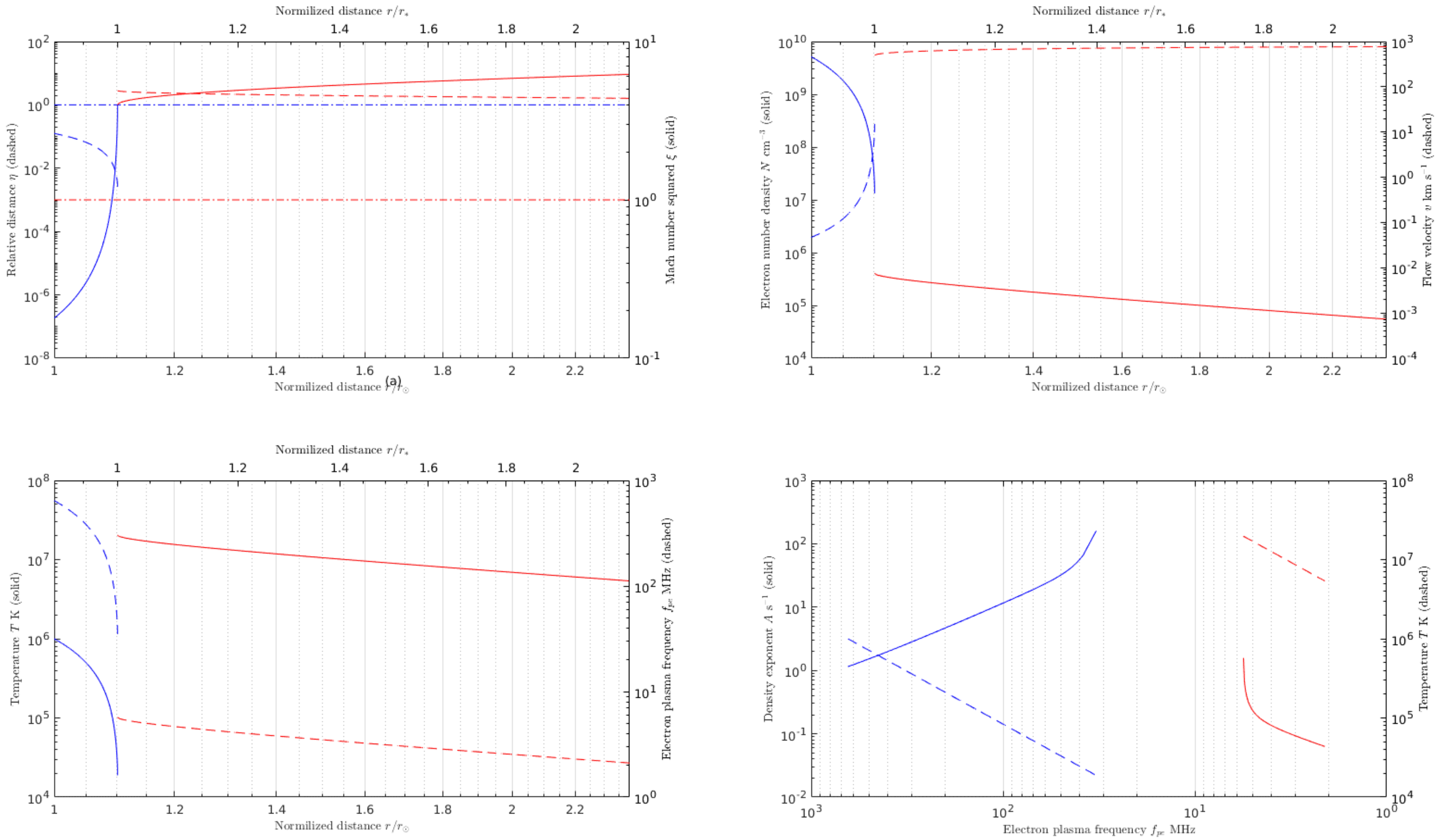


$$A = -\frac{1}{2n_e} \cdot \frac{dn_e}{dr} \cdot v_s.$$

Radio observations can be used to tune the solar wind patterns

The horizontal blue lines give range of observed values of A (Dididze et al. 2017).
The red lines correspond to the range of observed temperatures.

We present sample solution corresponding to the fast wind pattern.



The critical point is at 1.1 solar radii, the velocity at 1AU is 800 km/s

In case of any α the system is also integrable and analytic solution is available:

$$\xi\eta^4 = \left(\frac{C^2}{C_s^2}\right)^{\frac{5-3\alpha}{\alpha-1}} = (C_1 r_c)^{\frac{5-3\alpha}{\alpha-1}},$$

$$\frac{d\xi}{dr} - \frac{d\ln\xi\eta^4}{dr} - (3+\xi)\frac{d\ln r_c}{dr} = -\frac{4r_c}{r^2},$$

$$\frac{2 - Dr_c(\alpha-1)}{\alpha-1}\eta^4 - 4\eta^3 + (C_1 r_c)^{\frac{5-3\alpha}{\alpha-1}} = 0$$

Again from this equation solution flow patterns can be built.

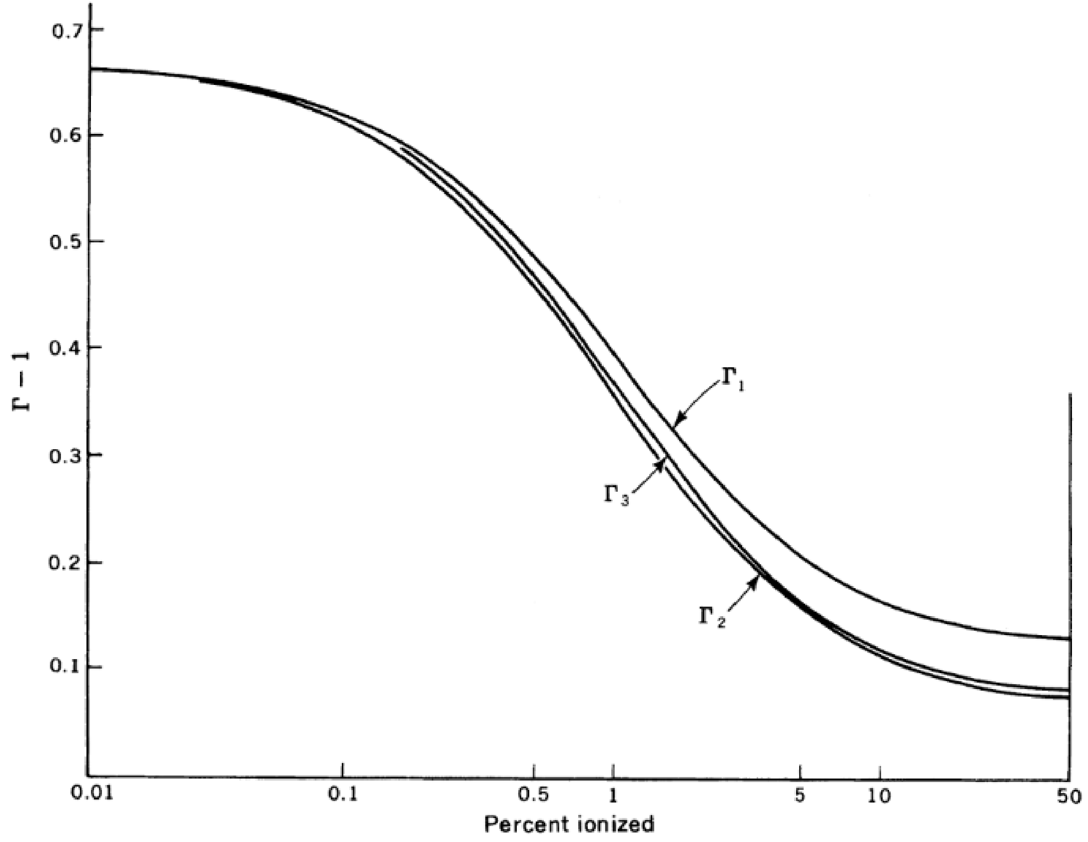


Fig. 2-13 The adiabatic exponents of a pure hydrogen gas as a function of its degree of ionization. Only the initial 50 percent is shown because the second 50 percent is its mirror image. The exponents change rapidly in the regions 0 to 1 or 99 to 100 percent ionization. Between 5 and 95 percent ionization, the values are considerably less than $\frac{4}{3}$ and therefore have a destabilizing influence on the structure. Numerical values are given in Table 2-4.

From Clyton (1968).

- ## Conclusions

- Flow profiles can be constructed for the modeling of the shear flow patterns.
- Using obtained solutions in combination with In-situ, remote-sensing and radio observations the source region can be more accurately be modelled and used for the solar wind numerical models and their space weather applications.
- The large scale flow pattern libraries can be developed for processing observational and numerical data mining, machine learning, regression and other artificial intelligence methods of the pattern (supervised) recognition, search and classification.

- ## Planned work

- To complete solution space analysis for the arbitrary value of the polytropic (adiabatic) index.
- To develop the complete set of procedures for the source region construction.
- To upgrade the model for the non-adiabatic cases when effects of transport processes and external forces are taken into account.

Thank you !